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NUMERICAL SIMULATION OF TURBULENT FLOW IN THE LAP OF A PAPER-MAKING MACHINE WITH A SEPARATING PLATE

V. E. Karyakin and Yu. E. Karyakin

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An algorithm is suggested for calculating viscous flows in the lap of a paper-making machine having a separating plate. The turbulent motion of liquid is described by the Reynolds equations with the use of the low-Reynolds-number k-ε-model. The results of calculations are presented.

At the present time, improvement of the elements of modern paper-making equipment is based to a large extent on the use of computational methods by means of which numerical simulation of viscous fluid flows in channels of arbitrary configuration is performed. A computational analysis permits one to examine in detail in a comparatively short time the structure of the flow and the complex effect on it of the governing parameters of the problem and also, by means of variation, to find the optimum and operational characteristics of the devices.

One of the important elements of a paper-making machine is its lap, i.e., the final converging channel in which the finishing preparation of the paper mass is made. Two, to a certain extent contradictory, requirements are imposed on the design of the lap: on the one hand, conditions should be provided for the best grinding and mixing of flocculas, which leads to the necessity of designing initial sections with a high level of turbulence; on the other hand, to obtain a high-quality product, the flow at the exit from the lap must be largely uniform with a very low level of turbulent fluctuations.

In the present work we extended the computational method suggested in [1, 2] to the case of the study of flow in a lap of complex geometry with a separating plate on the basis of the low-Reynolds-number k-ε-model of turbulence [3, 4].

The initial geometry of the lap is given in Fig. 1 (for case of graphical representation the transverse dimensions are increased fourfold; the fluid moves from right to left). The converging portion of the lap is divided into two parts by the separating plate, which additionally stabilizes the flow.

For numerical solution of the problem the method of splitting the computational domain into three subregions turned out to be most effective: 1 and 2 are, respectively, the lower and upper parts of the inlet channel up to the end of the plate, 3 is the exit part of the lap from the end of the plate to the outlet from the lap. In the first approximation use is made of the fact that upstream transfer of perturbations in the converging channel, which has a comparatively large hydraulic resistance, is rather weak. Therefore, solutions for the first two subregions can be found independently of one another, while the resulting distributions at the exit are the initial conditions for solving the problem in the third subregion. Next, iterative refinement of the problem is performed to take into account the upstream transfer of perturbations.

In each of the subregions we introduce a curvilinear nonorthogonal system of coordinates x^1, x^2 whose coordinate lines coincide with the boundaries of the channel. Let us direct the coordinate x^1 along the channel axis and x^2 in the transverse direction. We assume that the concentration of the paper mass is low, and then the working medium will behave like a Newtonian fluid. Its turbulent motion in the coordinate system x^1, x^2 will be described by the following Reynolds equations supplemented with the relations of the dissipative two-parametric low-Reynolds-number model of turbulence [3, 4] and presented in dimensionless tensor form:

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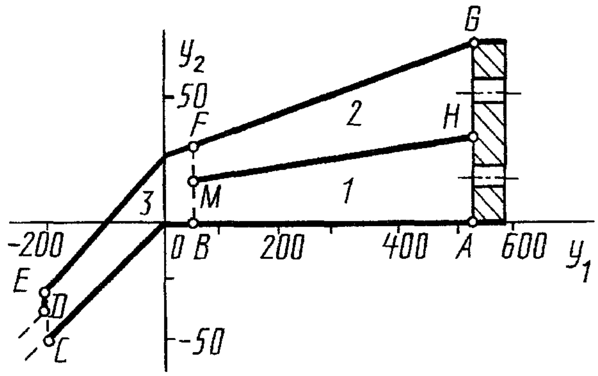


Fig. 1. Geometry of the lap of a paper-making machine with a separating plate.

$$\frac{\partial v_i}{\partial t} + \nabla_k (v^k v_i) + \nabla_i p_t = \nabla_k \left[\left(\frac{1}{\mathrm{Re}} + \nu_t \right) (g^{kl} \nabla_l v_i + g^{il} \nabla_l v_k) \right], \tag{1}$$

$$g^{kl} \nabla_k v_l = 0 \,, \tag{2}$$

$$\frac{\partial k}{\partial t} + \nabla_k (v^k k) = \nabla_k \left[\left(\frac{1}{\mathrm{Re}} + \frac{\nu_t}{\sigma_k} \right) g^{kl} \nabla_l k \right] + P - \varepsilon - \frac{1}{2\,\mathrm{Re}\,k} g^{kl} \nabla_k k \nabla_l k \,, \tag{3}$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla_k (v^k \varepsilon) = \nabla_k \left[\left(\frac{1}{\mathrm{Re}} + \frac{\nu_t}{\sigma_\varepsilon} \right) g^{kl} \nabla_l \varepsilon \right] + \frac{\varepsilon}{k} (C_1 P - C_2 f_\varepsilon \varepsilon) + Q \,, \tag{4}$$

where v_i, v^i ($i = 1, 2$) are the averaged co- and contravariant components of the velocity vector; p and $p = p + 2/3k$ are the ordinary hydrostatic and turbulent pressures; Δ_i is the symbol of the covariant derivative. Here and below, summation is performed from 1 to 2 over like indices, as is conventional in tensor analysis. The constants and functions entering into the system of equations (1)-(4) are defined by the following relations:

$$\nu_t = C_\mu f_\mu k^2/\varepsilon \,, \quad f_\mu = \exp\left(- b_1/(1 + b_2 \mathrm{Re}_t)\right) , \quad f_\varepsilon = 1 - a \exp\left(- \mathrm{Re}_t^2\right) ,$$

$$\mathrm{Re}_t = \mathrm{Re}\, k^2/\varepsilon \,, \quad C_\mu = 0.09 \,, \quad C_1 = 1.44 \,, \quad C_2 = 1.92 \,, \quad \sigma_k = 1 \,, \quad \sigma_\varepsilon = 1.3 \,,$$

$$a = 0.3 \,, \quad b_1 = 2.5 \,, \quad b_2 = 0.02 \,, \quad P = \frac{\nu_t}{2} g^{ik} g^{jl} (\nabla_i v_j + \nabla_j v_i)(\nabla_k v_l + \nabla_l v_k) \,,$$

$$Q = \frac{2}{\mathrm{Re}} \nu_t g^{il} g^{jm} g^{kn} (\nabla_i \nabla_j v_k)(\nabla_l \nabla_m v_n) \,.$$

With the use of transformations similar to those described in [1, 2], Eqs. (1)-(4) can easily be brought to a form not containing tensor derivatives:

$$\frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x^k} (\hat{v}^k \hat{v}_i) + \frac{\partial p_T}{\partial x^i} = \frac{\partial}{\partial x^k} \left[\left(\frac{1}{\mathrm{Re}} + \nu_t \right) \hat{g}^{kl} \frac{\partial \hat{v}_i}{\partial x^l} \right] + \frac{\partial \nu_t}{\partial x^k} \frac{\partial \hat{v}^k}{\partial x^i} , \tag{5}$$

$$g^{kl} \frac{\partial \hat{v}_l}{\partial x^k} = 0 \,, \tag{6}$$

$$\frac{\partial k}{\partial t}+\frac{\partial}{\partial x^k}(\hat{v}^k k)=\frac{\partial}{\partial x^k}\left[\left(\frac{1}{\mathrm{Re}}+\frac{\nu_t}{\sigma_k}\right)\hat{g}^{kl}\frac{\partial k}{\partial x^l}\right]+P-\varepsilon-\frac{1}{2\,\mathrm{Re}\,k}g^{kl}\frac{\partial k}{\partial x^k}\frac{\partial k}{\partial x^l}, \tag{7}$$

$$\frac{\partial \varepsilon}{\partial t}+\frac{\partial}{\partial x^k}(\hat{v}^k \varepsilon)=\frac{\partial}{\partial x^k}\left[\left(\frac{1}{\mathrm{Re}}+\frac{\nu_t}{\sigma_\varepsilon}\right)\hat{g}^{kl}\frac{\partial \varepsilon}{\partial x^l}\right]+\frac{\varepsilon}{k}(C_1P-C_2f_\varepsilon\,\varepsilon)+Q. \tag{8}$$

The symbol (^) was used to denote quantities calculated by means of matrices of the derivatives $\partial x^i/\partial y_\alpha$ and $\partial y_\alpha/\partial x^i$ fixed at the point of differentiation c [2]:

$$\hat{v}_i=u_\alpha\,(\partial y_\alpha/\partial x^i)_c,\quad \hat{v}^i=u_\alpha\,(\partial x^i/\partial y_\alpha)_c,\quad \hat{g}^{kl}=\partial x^l/\partial y_\alpha\,(\partial x^k/\partial y_\alpha)_c.$$

In each of the subregions the system of equations (5)-(8) is supplemented with corresponding boundary conditions. At the entrance to the computational domain both velocity components v_1 and v_2 as well as the kinetic energy of turbulent fluctuations k and the rate of its dissipation ε as a function of the transverse coordinate x^2 are prescribed. These functions are either known a priori (the boundaries *AH* and *HG*) or are determined from the calculation of the flow in conjugate subregions (the boundary *BF* for subregion 3). On the solid walls (*AB*, *MH*, *FG*, *BC*, and *FE*) the no-slip condition and the condition of the turbulent characteristics k and ε being equal to zero are written: $v_1=v_2=k=\varepsilon=0$.

In this problem the boundary conditions on the exit boundaries of the subregions have some distinctive features. In the first iteration the calculation is performed with modified weak boundary conditions similar to those described in [1]:

$$\frac{\partial}{\partial x^1}v_j\left(\frac{\partial x^j}{\partial y_1}\frac{\partial y_2}{\partial x^2}-\frac{\partial x^j}{\partial y_2}\frac{\partial y_1}{\partial x^2}\right)=0,\quad \frac{\partial v_2}{\partial x^2}=\frac{\partial k}{\partial x^2}=\frac{\partial \varepsilon}{\partial x^2}=0. \tag{9}$$

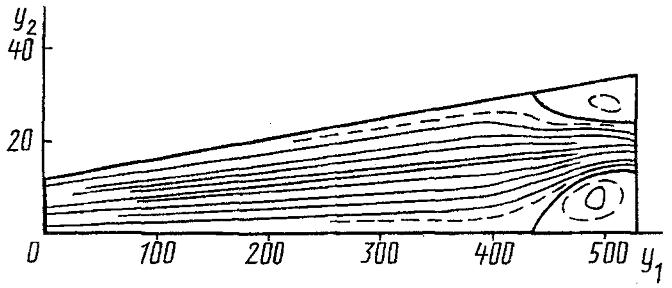
Hereafter, conditions (9) are invariant only near the boundary *CD*, whereas on the exit boundaries *BM* and *MF* of subregions 1 and 2 they are changed in view of the inverse effect upstream:

$$\frac{\partial}{\partial x^1}v_j\left(\frac{\partial x^j}{\partial y_1}\frac{\partial y_2}{\partial x^2}-\frac{\partial x^j}{\partial y_2}\frac{\partial y_1}{\partial x^2}\right)=f_1(x^2),\quad \frac{\partial v_2}{\partial x^2}=f_2(x^2),$$

$$\frac{\partial k}{\partial x^2}=f_3(x^2),\quad \frac{\partial \varepsilon}{\partial x^2}=f_4(x^2),$$

where the functions $f_1(x^2)$, $f_2(x^2)$, $f_3(x^2)$, and $f_4(x^2)$ represent combinations that correspond to the left-hand parts and are calculated from the values of the flow parameters in the previous iteration along the boundary *BF* of subregion 3. A numerical experiment showed that such a procedure for taking into account the inverse effect gives rather good convergence.

When solving system of equations (5)-(8), use is made of a multistep implicit difference scheme described in [2] and [5] and extended to the case of a turbulent flow. The unknown grid functions v_1, v_2, p, k, and ε are determined on grids shifted relative to one another, as is conventional in the marker-and-mesh method. To approximate convective terms use is made of the Leonardo scheme [6] of quadratic interpolation upstream, and the remaining terms are approximated by central differences. Finite-difference equations are written for the corrections to the values sought. To accelerate the convergence of the iterative process of finding the field of pressures a set of relaxation time steps is used that ensures uniform decay of the error in the entire spectrum of the natural frequencies of the problem. Splitting of the equations for the corrections ($i=1,2$), δk, and $\delta\varepsilon$ over space variables in the implicit stages of the algorithm is performed directly during solution of the problem and is determined by the direction of the flow. The resulting difference scheme acquires parabolic properties; it is especially effective in



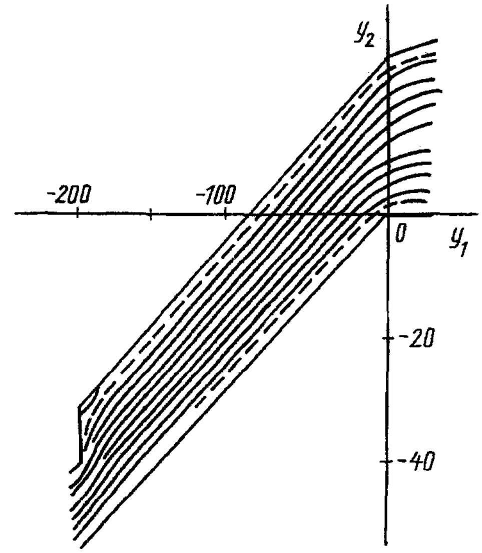


Fig. 2. Streamlines in subregion 1 of the lap.

Fig. 3. Streamlines in subregion 3 of the lap.

studying flows in which circulatory zones of reverse motion exist against the background of the characteristic predominant direction of the flow.

In connection with the strong nonlinearity of Eqs. (7) and (8) their difference analogs are considered as a single vector equation whose solution is achieved by means of vector elimination. The source term of this equation is linearized by Newton's method. Moreover, to perform iterations use is made of linear stabilization, which excludes the possibility of the appearance of nonphysical negative values of the quantities k and ε.

Using the above-described difference algorithm we performed numerical simulation of turbulent flow in the lap of a paper-making machine (Fig. 1) at different values of the operational and geometric parameters. The nominal width of the exit slit CD is equal to 12.7 mm. The paper mass is supplied through 16-mm-diameter holes located in two rows with a spacing of 28 mm. For the calculations the channel was extended to a certain distance where conditions for smooth behavior of the unknown functions are written down. The following are some results of the calculations.

In Fig. 2 the picture of streamlines in subregion 1 is presented for the nominal regime of operation of the lap with $Re = 4.1 \cdot 10^4$ and the length of the separating plate equal to 500 mm. At the entrance to the channel (on the right) a stepwise velocity profile is prescribed corresponding to the inflow of the paper mass through a hole, while the characteristics of the turbulence are determined by means of an algebraic model. The streamlines in Fig. 2 are depicted as solid lines with the dimensionless flowrate step 0.1. Where possible, the dashes show additional isolines. It is seen that near the entrance at a length of the order of 100 mm the paper mass jet gradually expands and intense circulating flows form. Imperfect symmetry of the supply of mass determines a noticeable asymmetry of the flow. This asymmetry decays very rapidly downstream and becomes completely imperceptible near the exit. Some flow retardation near the walls is noted. The effect of the exit region on the flow is small.

In Fig. 3 a picture of streamlines in the exit subregion 3 is given for the same regime and geometry. Solid isolines are drawn with the step 0.1 but on the basis of a total flowrate that is equal to the sum of the flowrates through regions 1 and 2. It is seen that the effect of the separating plate in the middle portion of the channel extends over a rather appreciable distance. A detailed analysis shows that the flow preserves its asymmetry almost up to the exit from the lap. At the same time at the exit CD the flow is rather homogeneous and is almost parallel to the wall. We note that in the vicinity of the exit near the sharp edge a rather strong increase in the intensity of the turbulence up to $k = 0.1$ occurs, which favors the destruction of the paper mass flocculas. The characteristics of the turbulence at the exit depend weakly on the length of the separating plate. At the same time, the coefficient of hydraulic losses of the lap has a rather clear minimum for the length of the plate equal to 500 mm.

Thus, computational investigations showed that a computational experiment can be used successfully both for simulation of turbulent flows in the lap of a paper-making machine and for finding its optimum shape at which the deflocculating effect of turbulent pulsations on the paper mass flow increases.

NOTATION

y_1, y_2, Cartesian coordinates; x^1, x^2, curvilinear coordinates; u_1, u_2, Cartesian velocity components; v_1, v_2 and v^1, v^2, co- and contravariant velocity components; g^{kl}, metric tensor components; k, kinetic energy of turbulent fluctuations; ε, rate of dissipation of the energy of turbulent pulsations; t, time; ν_t, coefficient of turbulent viscosity; Re, Reynolds number; δ, increment of functions.

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